SECTION – A

Questions 1 to 10 carry 1 mark each.

1. If m and n are the order and degree, respectively of the differential equation $\dfrac{d}{dx}\left[\left(\dfrac{dy}{dx}\right)^{4}\right]=0$,

   then write the value of m + n.

   (a) 1   (b) 2   (c) 3   (d) 4

   Ans: (c) 3

   $\dfrac{d}{dx}\left[\left(\dfrac{dy}{dx}\right)^{4}\right]=0 \Rightarrow 4\left(\dfrac{dy}{dx}\right)^{3}\dfrac{d^{2}y}{dx^{2}}=0$

   Here, m = 2 and n = 1

   Hence, m + n = 3

2. The number of solutions of the differential equation $\dfrac{dy}{dx}=\dfrac{y+1}{x-1}$, when y(1) = 2 is

   (a) one   (b) one   (c) two   (d) infinite

   Ans. (b) one

   Given that; $\dfrac{dy}{dx}=\dfrac{y+1}{x-1} \Rightarrow \dfrac{dy}{y+1}=\dfrac{dx}{x-1}$

   On integrating both sides, we get $\displaystyle\int\dfrac{dy}{y+1}=\int\dfrac{dx}{x-1}$

   $\Rightarrow$ log(y + 1) = log(x – 1) – logC

   $\Rightarrow$ log(y + 1) + logC = log (x – 1) $\Rightarrow$ $C=\dfrac{x-1}{y+1}$

   Now, $y(1) = 2 \Rightarrow C = \dfrac{1-1}{2+1} = 0$

   $\therefore$ Required solution is x – 1 = 0

   Hence, only one solution exist.

3. The number of arbitrary constants in the particular solution of a differential equation of second order is (are):

   (a) 0   (b) 1   (c) 2   (d) 3

   Ans. (a) 0

   Particular solution of a differential equation of any order, does not have any arbitrary constant.

4. Kapila is trying to find the general solution of the following differential equations.

   (I) $xe^{x/y}dx - ye^{3x/y}dy = 0$

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(II) $(2x+1)\dfrac{dy}{dx} = 3 - 2yc$

(III) $\dfrac{dy}{dx} = \sin x - \cos y$

Which of the above become variable separable by substituting y = b.x, where b is a variable?
(a) only (I) (b) only (I) and (II) (c) all - (I), (II) and (III) (d) None of these
Ans. (a) only (I)
The homogeneous differential equation become variable separable by substituting y = bx.
Among given equations. only (I) is a homogeneous differential equation.

5. The general solution of the differential equation ydx – xdy = 0; (Given x, y > 0), is of the form:
   (a) xy = c   (b) x = cy²   (c) y = cx   (d) y = cx²
   (Where 'c' is an arbitrary positive constant of integration)
   Ans. (c) y = cx

6. The solution of the differential equation $2x\dfrac{dy}{dx} - y = 3$ represents:

   (a) a circle   (b) an ellipse   (c) a straight line   (d) a parabola
   Ans. (d) a parabola
   We have, $2x\dfrac{dy}{dx} - y = 3$

   $\Rightarrow 2x\dfrac{dy}{dx} = 3 + y \Rightarrow 2\dfrac{dy}{3+y} = \dfrac{dx}{x}$

   On integrating both sides, we get
   2 log (y + 3) = log x + log c
   $\Rightarrow$ log (y + 3)² = log cx
   $\Rightarrow$ (y + 3)² = cx

7. The order and the degree of the differential equation $3x^2\left(\dfrac{d^2y}{dx^2}\right)^3 - 3\left(\dfrac{dy}{dx}\right)^4 + y = 0$ are:

   (a) 2, 1   (b) 2, 3   (c) 2, 4   (d) 3, 1
   Ans: (b) 2, 3
   The highest order is 2 and the degree of the highest order is 3.
   Hence, the order is 2 and the degree is 3.

8. Integrating factor for the differential equation $(x \log x)\dfrac{dy}{dx} + y = 2 \log x$ is

   (a) log (log x)   (b) log x   (c) eˣ   (d) x
   Ans. (b) log x
   Equation is $\dfrac{dy}{dx} + \dfrac{1}{x\log x}\cdot y = \dfrac{2}{x}$

   Here $P(x) = \dfrac{1}{x\log x}$

   Integrating factor $= e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

In the following questions 9 and 10, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

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9. **Assertion (A):** The solution of differential equation $\dfrac{dy}{dx} = \dfrac{y}{x}$ with initial condition x = 1 and y =

1 is x = y.

Reason (R): Separation of variable method can be used to solve the differential equation.

Ans. (a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A).

10. **Assertion (A):** Solution of the differential equation $(1 + x^2)\dfrac{dy}{dx} + y = \tan^{-1} x$ is

$ye^{\tan^{-1} x} = (\tan^{-1} x - 1)e^{\tan^{-1} x} + C$

Reason (R) : The differential equation of the form $\dfrac{dy}{dx} + Py = Q$, where P, Q be the functions

of x or constant, is a linear type differential equation.

Ans: (b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A).

SECTION – B
Questions 11 to 14 carry 2 marks each.

11. Find the general solution of the following differential equation: $e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$.

Ans.

Given differential equation,

$e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0 \;\Rightarrow\; e^x \tan y\, dx = -(1 - e^x)\sec^2 y\, dy$

$\Rightarrow \quad \dfrac{\sec^2 y\, dy}{\tan y} = \dfrac{e^x}{e^x - 1}dx$

Integrating both sides, we get

$\displaystyle\int \dfrac{\sec^2 y\, dy}{\tan y} = \int \dfrac{e^x}{e^x - 1}dx \quad \Rightarrow\; \log|\tan y| = \log|e^x - 1| + \log C$

$\Rightarrow \quad \log|\tan y| = \log|(e^x - 1)\,C|$

$\therefore \quad \tan y = (e^x - 1)\,C$

12. Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$

Ans: The given differential equation is $x^2(1 - y)\,dy + y^2(1 + x^2)\,dx = 0$

$\Rightarrow \; x^2(1 - y)dy = -y^2(1 + x^2)dx$

$\Rightarrow \; \dfrac{1-y}{y^2}dy = -\left(\dfrac{1+x^2}{x^2}\right)dx$, if $x, y \neq 0$

$\Rightarrow \; \left(\dfrac{1}{y} - \dfrac{1}{y^2}\right)dy = \left(\dfrac{1}{x^2} + 1\right)dx \;\Rightarrow\; \int\left(\dfrac{1}{y} - \dfrac{1}{y^2}\right)dy = \int\left(\dfrac{1}{x^2} + 1\right)dx$

$\Rightarrow \; \log|y| + \dfrac{1}{y} = -\dfrac{1}{x} + x + C$, which is the general solution of the differential equation.

13. Find the general solution of the differential equation $\dfrac{dy}{dx} = x - 1 + xy - y$.

Ans: $\dfrac{dy}{dx} = x - 1 + xy - y = (x-1)(y+1)$

$\Rightarrow \int \dfrac{dy}{y+1} = \int (x-1)\,dx$

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$\Rightarrow \log|y+1| = \dfrac{x^2}{2} - x + C$ is the required solution.

14. Solve the following differential equation: $\dfrac{dy}{dx} = x^3 \cos ecy$, given that $y(0) = 0$.

Ans: Given differential equation: $\dfrac{dy}{dx} = x^3 \cos ecy$,

$\therefore \quad \dfrac{dy}{\cos ec\ y} = x^3.dx$

Integrating both sides, we get $\int \sin y\ dy = \int x^3\ dx$

$\Rightarrow -\cos y = \dfrac{x^4}{4} + c \qquad\qquad \Rightarrow \quad -\cos y = \dfrac{0}{4} + c$

$\Rightarrow -1 = c$

Putting the value of c in (i), we get

$-\cos y = \dfrac{x^4}{4} - 1 \qquad\qquad \therefore \quad \cos y = \left(1 - \dfrac{x^4}{4}\right)$

SECTION – C
Questions 15 to 17 carry 3 marks each.

15. For the differential equation $xy\dfrac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through the point $(1, -1)$.

Ans.

The given equation is $xy\dfrac{dy}{dx} = (x+2)(y+2) \quad\Rightarrow\quad \dfrac{y}{y+2}dy = \dfrac{x+2}{x}dx$

$\Rightarrow \qquad \int\dfrac{y}{y+2}dy = \int\left(\dfrac{x+2}{x}\right)dx \qquad\Rightarrow\quad \int\left(1 - \dfrac{2}{y+2}\right)dy = \int\left(1 + \dfrac{2}{x}\right)dx$

$\Rightarrow\quad y - 2\log|y+2| = x + 2\log|x| + C$

$\Rightarrow\quad y = x + 2\log|x| + 2\log|y+2| + C$

$\Rightarrow\quad y = x + 2\log|x(y+2)| + C$

Since the line passes through the point $(1, -1)$. So, putting $x = 1, y = -1$.

We have, $\ -1 = 1 + 2\log|1(-1+2)| + C \Rightarrow C = -2$

$\therefore \qquad\qquad y = x + 2\log|x(y+2)| - 2$, which is the required equation of the curve.

16. Find the particular solution of the differential equation: $(1 + e^{2x})dy + (1 + y^2)e^x\ dx = 0$ given that $y = 1$ when $x = 0$.

Ans.

We have, $(1 + e^{2x})\ dy + (1 + y^2)\ e^x\ dx = 0$ and given that $y = 1$, when $x = 0$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{-(1+y^2)e^x}{1+e^{2x}} \qquad\Rightarrow\quad \dfrac{dy}{-(1+y^2)} = \dfrac{e^x dx}{1+e^{2x}}$

Integrating both sides, we get

$-\int\dfrac{dy}{1+y^2} = \int\dfrac{e^x dx}{1+e^{2x}} \qquad\Rightarrow -\tan^{-1}y = \int\dfrac{e^x dx}{1+(e^x)^2}$

$\Rightarrow -\tan^{-1}y = \int\dfrac{dt}{1+t^2} \qquad$ [Putting $e^x = t \Rightarrow e^x dx = dt$]

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$\Rightarrow -\tan^{-1} y = \tan^{-1}(t) + C \quad \Rightarrow -\tan^{-1} y = \tan^{-1}(e^x) + C$

Put $x = 0, y = 1$ in (i), we get

$-\tan^{-1} 1 = \tan^{-1}(e^0) + C \qquad \Rightarrow -\dfrac{\pi}{4} = \dfrac{\pi}{4} + C \Rightarrow C = -\dfrac{\pi}{2}$

Putting the value of C in (i), we get

$-\tan^{-1} y = \tan^{-1}(e^x) - \dfrac{\pi}{2} \qquad \Rightarrow \dfrac{\pi}{2} = \tan^{-1}(e^x) + \tan^{-1} y$

Hence, $\tan^{-1}(e^x) + \tan^{-1} y = \dfrac{\pi}{2}$ is the required solution.

17. Solve the following differential equation: $\dfrac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \dfrac{\pi}{3}$.

Ans. Given differential equation is $\dfrac{dy}{dx} + 2y\tan x = \sin x$

Comparing it with $\dfrac{dy}{dx} + Py = Q$, we get $P = 2\tan x, Q = \sin x$

$\therefore \quad$ IF $= e^{\int 2\tan x\, dx} = e^{2\log\sec x} = e^{\log\sec^2 x} = \sec^2 x \qquad [\because e^{\log z} = z]$

Hence, general solution is $y.\sec^2 x = \int \sin x . \sec^2 x\, dx + C$.

$y.\sec^2 x = \int \sec x . \tan x\, dx + C \Rightarrow y.\sec^2 x = \sec x + C \Rightarrow y = \cos x + C\cos^2 x$

Putting $y = 0$ and $x = \dfrac{\pi}{3}$, we get $0 = \cos\dfrac{\pi}{3} + C.\cos^2\dfrac{\pi}{3}$

$\Rightarrow 0 = \dfrac{1}{2} + \dfrac{C}{4} \quad \Rightarrow \quad C = -2$

$\therefore \quad$ Required solution is $y = \cos x - 2\cos^2 x$.

SECTION – D
Questions 18 carry 5 marks.

18. Solve the following differential equation: $\left(1 + e^{y/x}\right)dy + e^{y/x}\left(1 - \dfrac{y}{x}\right)dx = 0, (x \neq 0)$

Ans. Given differential equation $\left(1 + e^{y/x}\right)dy + e^{y/x}\left(1 - \dfrac{y}{x}\right)dx = 0$

$\Rightarrow (1 + e^{y/x})dy = \left(\dfrac{y}{x} - 1\right)e^{y/x}dx \Rightarrow \dfrac{dy}{dx} = \dfrac{\left(\dfrac{y}{x} - 1\right)e^{y/x}}{(1 + e^{y/x})}$

It is a homogeneous differential equation.

Put $\ y = vx \ $ and $\ \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

We have,

$v + x\dfrac{dv}{dx} = \dfrac{(v - 1)}{1 + e^v}e^v = \dfrac{ve^v - e^v}{1 + e^v}$

$\Rightarrow x\dfrac{dv}{dx} = \dfrac{ve^v - e^v}{1 + e^v} - v = \dfrac{ve^v - e^v - v - ve^v}{1 + e^v}$

$\Rightarrow x\dfrac{dv}{dx} = -\dfrac{(v + e^v)}{1 + e^v} \Rightarrow \dfrac{1 + e^v}{v + e^v}dv = -\dfrac{dx}{x}$

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On integrating both sides, we have

$$\int \frac{1+e^v}{v+e^v}dv = -\int \frac{dx}{x}$$

$\Rightarrow \quad \log|v+e^v| = -\log|x| + \log|C|$

$\Rightarrow \quad \log|v+e^v| + \log|x| = \log|C|$

$\Rightarrow \quad \log|x(v+e^v)| = \log|C|$

$\Rightarrow \quad x(v+e^v) = C \Rightarrow x\left(\frac{y}{x} + e^{y/x}\right) = C \Rightarrow y + x\,e^{y/x} = C$

SECTION – E (Case Study Based Questions)
Questions 19 to 20 carry 4 marks each.

19. **Case-Study 1: Read the following passage and answer the questions given below.**

As per the Newton's law of cooling, the rate at which an object cools is given by the following

equation: $\dfrac{d\theta}{dt} = -k(\theta - \theta_s)$

where $\theta = \theta(t)$ is temperature of cooling object at time, t, θ_s is the temperature of the environment (assumed to be constant) and k is the thermal constant related to the cooling object.
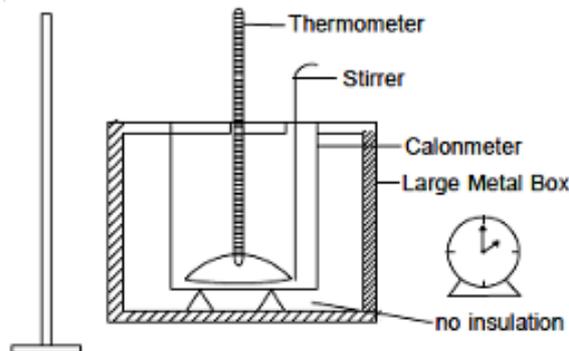
(a) Write the order and degree of the above given differential equation.

(b) Find the solution of the given differential equation: $\dfrac{d\theta}{dt} = -k(\theta - \theta_s)$

(c) If $\theta = \theta_0$ (initial temperature of cooling object) at time t = 0, then find the particular solution of the above given differential equation.

OR

Find the rate of cooling of the object at 50°C, if the thermal constant is $\dfrac{3}{25}$ min^{-1} and temperature of surroundings is 25°C.

Ans: (a) $\dfrac{d\theta}{dt} = -k(\theta - \theta_s)$

highest order derivative is $\dfrac{d\theta}{dt}$ so order is 1. The exponent of highest order derivative is 1. so its degree is 1.

(b) $\dfrac{d\theta}{dt} = -k(\theta - \theta_s) \Rightarrow \dfrac{d\theta}{(\theta - \theta_s)} = -kdt$

$\Rightarrow \int \dfrac{d\theta}{(\theta - \theta_s)} = -k\int dt$

$\Rightarrow \log_e(\theta - \theta_s) = -kt + C$

$\Rightarrow \theta - \theta_s = e^{-kt+c}$

$\Rightarrow \theta = e^{-kt+c} + \theta_s$

(iii) As $\theta = e^{-kt+C} + \theta_s$

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At $t = 0$, $\theta = \theta_0$, then
$\theta_0 = e^c + \theta_s \Rightarrow e^c = \theta_0 - \theta_s$
So, particular solution is,
$\theta = e^{-kt}(\theta_0 - \theta_s) + \theta_s$

<div align="center">OR</div>

(c) $\dfrac{d\theta}{dt} = -k(\theta - \theta_s)$

$\Rightarrow \dfrac{d\theta}{dt} = -\dfrac{3}{25}(50 - 25) = -3^0\,C/\min$

So, rate of cooling is 3°C/min Case-Based

20. Case-Study 2: Read the following passage and answer the questions given below.

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product' of the rate of bank interest per annum and the principal. In other words, compound interest is the interest calculated on the principal and the interest accumulated over the previous period. It is different from simple interest, where interest is not added to the principal while calculating the interest during the next period. Let P denotes the principal at any time t and rate of interest be r % per annum.

Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\dfrac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

(a) Find the solution of the differential equation $\dfrac{dy}{dx} = k(50 - y)$ (1)

(b) Find the value of c in the particular solution given that $y(0) = 0$ and $k = 0.049$ (1)

(c) Find the solution that may be used to find the number of children who have been given the polio drops. (2)

Ans: (a) We have, $\dfrac{dy}{dx} = k(50 - y)$

$\int \dfrac{dy}{50 - y} = \int k\,dx \Rightarrow -\log|50 - y| = kx + C$

(b) Given $y(0) = 0$ and $k = 0.049$
$-\log|50 - y| = kx + C$
$\Rightarrow -\log|50 - 0| = 0.049\,(0) + C$
$\Rightarrow -\log 50 = C \Rightarrow C = \log\dfrac{1}{50}$

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(c) We have, $-\log|50-y| = kx + \log\dfrac{1}{50}$ [From (a) and (b)]

$\Rightarrow -kx = \log|50-y| + \log\dfrac{1}{50} \Rightarrow -kx = \log\dfrac{50-y}{50} \Rightarrow e^{-kx} = \dfrac{50-y}{50} = 1 - \dfrac{y}{50}$

$\Rightarrow \dfrac{y}{50} = 1 - e^{-kx} \Rightarrow y = 50(1 - e^{-kx})$

This is the required solution to find the number of children who have been given the polio drops.

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